CHAPTER 7

STRESSES IN SOIL

عندما تجرى فى التربة اعمال حفر او انشاء ينشأ عن ذلك اجهادات تنتشر (عمليا) فى منطقة معينة من التربة ويؤدى ذلك الى حدوث انفعالات فى هذة المنطقة وتتأثر الاعمال التى تقام بمحصلة انفعالات التربة فى المنطقة المتأثرة بالاجهادات. لذلك فانه عند دراسة ميكانيكا التربة ومدى تأثيرها على سلامة وامان اعمال الحفر او المنشات يلزم حساب انتشار الاجهادات فى التربة.

باستخدام نظرية المرونة تمكن الرياضيون ورائدهم الفرنسى بوزينيسك (١٨٨٥) من حساب الاجهادات فى التربة تحت حالات مختلفة من التحميل. وهناك طرق مختلفة تأخذ فى الاعتبار عدم تجانس التربة واختلاف خواص المرونة لها رأسيا وأفقيا واختلاف الطبقات. ويعنى هذا الباب بحساب الاجهادات الرأسية فى اى نقطة فى التربة باعتبارها جسم متجانس مرن نصف لانهائى يتبى قانون هوك نتيجة احمال رأسيا مركزة او موزعة طوليا او على مساحة مرنة معينة. اما بالنسبة للاجهادات الافقية فى الاتجاهين او الاجهادات نتيجة الاحمال داخل التربة او الموزعة على مساحات صلدة فلها معادلات اخرى يمكن معرفتها من المراجع التخصصية.

The knowledge of the way in which foundation loads are transmitted to the soil supporting a foundation, and the extent, and distribution of the stresses created within the soil mass, are of great importance to the design engineer. Such a knowledge enables estimates to be made for possible settlement and bearing capacity, and permits the planning of adequate site exploration, besides providing data for dimensioning of foundations. Most of the methods used for studying stress distribution within soil mass are based on the elastic theory. The following assumptions are made:

(a) Soil is semi-infinite in extent.
(b) Soil is isotropic.
(c) Soil is homogeneous.
(d) Soil is elastic and obeys Hook's law.

Natural soils are seldom fulfilling these conditions, but practically actual settlements and pressures are in reasonable agreement with the "theoretical" calculated values.

7.1 Stresses due to concentrated load (Boussinesq equation)

For vertical concentrated load acting on the horizontal surface of soil, Fig. 7.1, the stresses have circular symmetry about the vertical line through the point of application of the load. The vertical stress is given by:

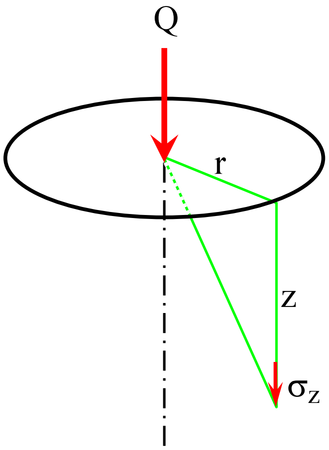


Fig 7.1 Effect of concentrated load

$$\sigma_z = \frac{3\,Q}{2\,\pi\,z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \qquad \text{.. .. .. (7.1)}$$

Put: $$N_B = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \qquad \text{.. .. ..} \quad (7.2)$$

$$\boxed{\sigma_z = \frac{Q}{z^2}\,.\,N_B} \qquad \text{.. .. ..} \quad (7.3)$$

Values of N_B can be obtained from Fig. 7.2.

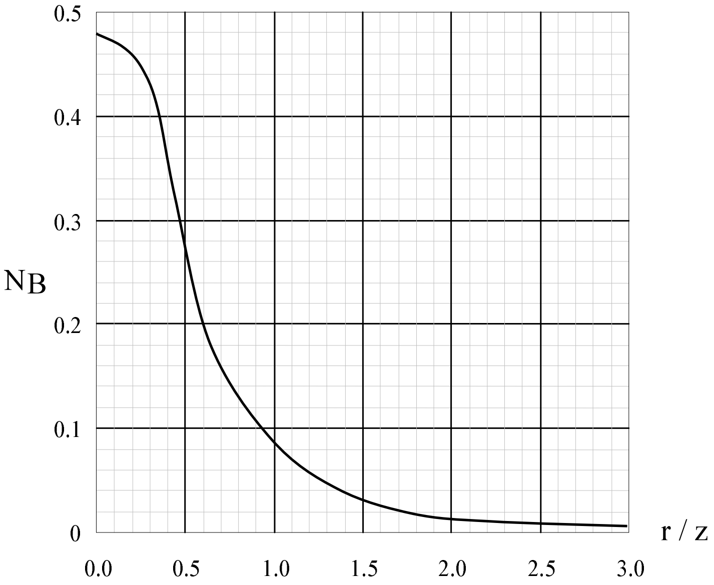


Fig 7.2 Solution of Boussinesq equation for concentrated load

7.2 Vertical stress due to line load

The vertical stresses at a point defined by x, y, z are given by:

$$\sigma_z = \frac{q}{z}\,.\,N_L \qquad \text{.. .. ..} \quad (7.4)$$

q = Intensity of line load.
x = Horizontal distance from plane of load.
y = Distance from end of line load.
z = Depth below surface.
N_L = Dimensionless influence factor.

Values of N_L are obtained from Fig. 7.3 by determining the ratios $m = x / z$, and $n = y / z$.

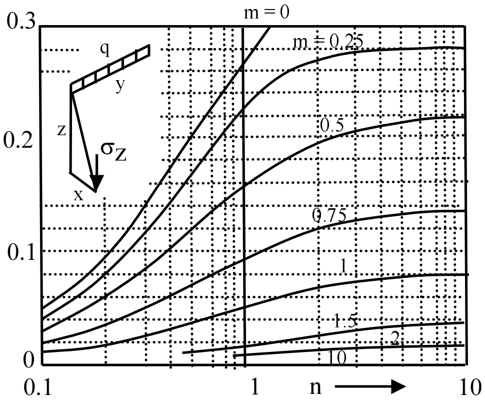


Fig 7.3 Stresses due to line load

7.3 Vertical stress under a uniformly loaded rectangular area:

For a uniform load of intensity (q), the stress under the corner of a rectangle of sides a x b, at depth z is given by:

$$\sigma_z = q \cdot N_r \qquad \text{.. .. .. (7.5)}$$

Where N_r is obtained for values of $m = L / b$, and $n = z / b$ as given in Fig 7.4.

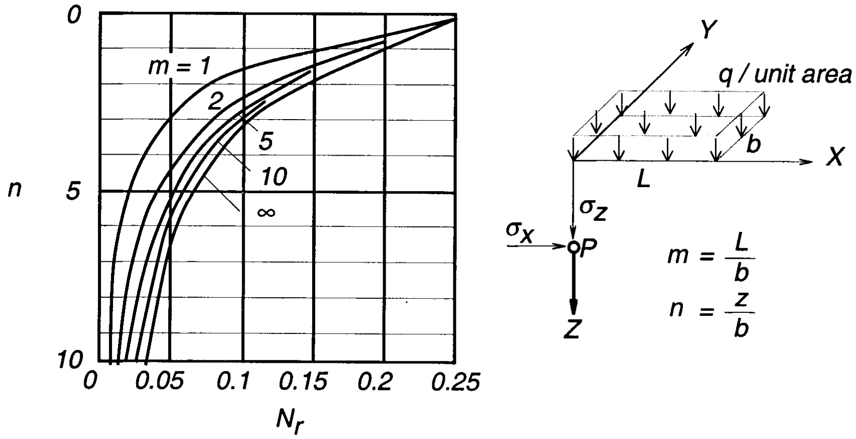


Fig 7.4 Stresses due to rectangular loaded area

For point p, not located at the corner of the loaded area ABCD, the following cases are considered, Fig. 7.5:

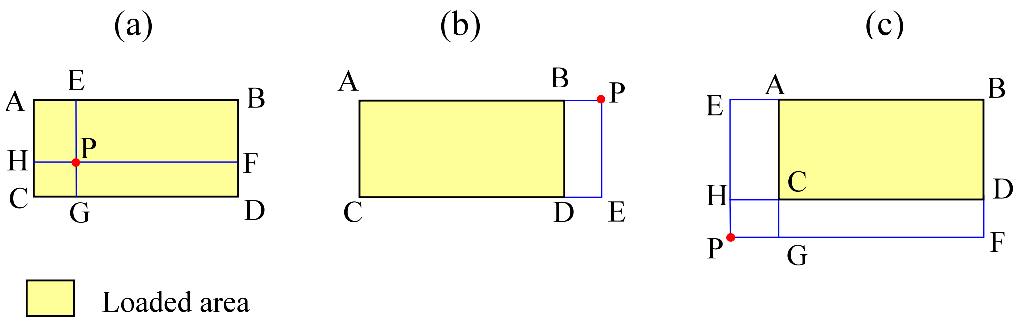


Fig 7.5 Calculation of stress at point not at the corner of the loaded area

Case (a):

$$\sigma_z = \sigma_z(AEPH) + \sigma_z(EBFP) + \sigma_z(FDGP) + \sigma_z(GCHP)$$

Case (b):

$$\sigma_z = \sigma_z(APEC) - \sigma_z(BPED)$$

Case (c):

$$\sigma_z = \sigma_z(EBFP) - \sigma_z(EAGP) - \sigma_z(HDFP) + \sigma_z(HCGP)$$

7.4 Newark Chart

The Newmark chart can be used to calculate the vertical stress due to loaded area of any shape under a point located any where inside or outside the loaded area. The Newmark chart, Fig. 7.6, consists of a number of circles and radiating lines. A distance AB is given on the chart, together with a corresponding influence value (0.005).

7.4.1 Procedure

(a) Draw the shape of the loaded area ABCD, Fig. 7.7, on a drawing paper to scale AB = z, and locate point P under which the stress is to be calculated.

(b) Placed point P in the drawing paper above the center of the circle of the Newmark chart, and count the number of divisions inside the loaded area (N).

(c) Calculate (σ_z) from the Equation

$$\sigma_z = 0.005\ q\,.\,N \qquad \text{.. .. .. (7.6)}$$

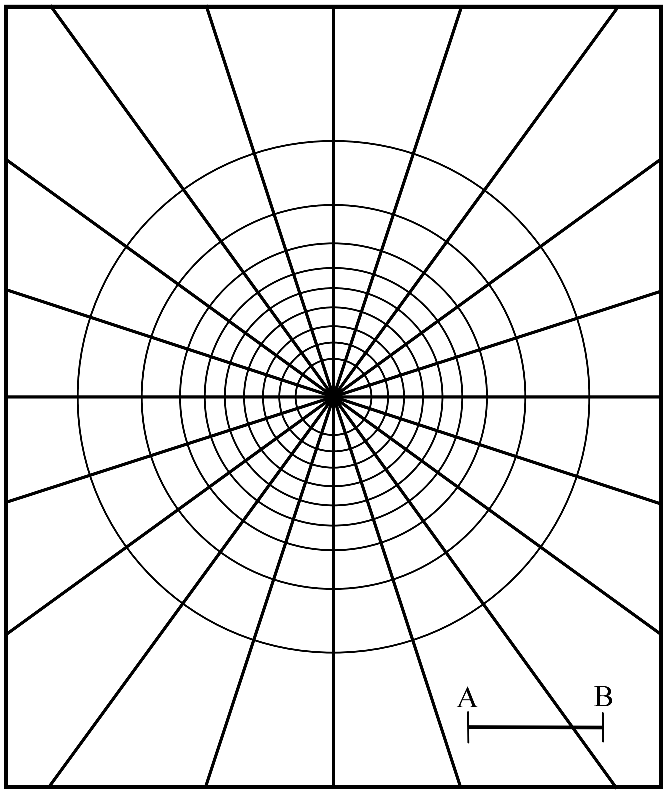


Fig 7.6 Newmark chart

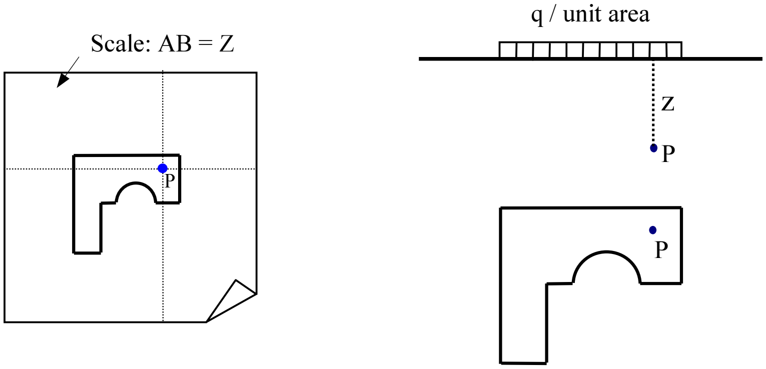
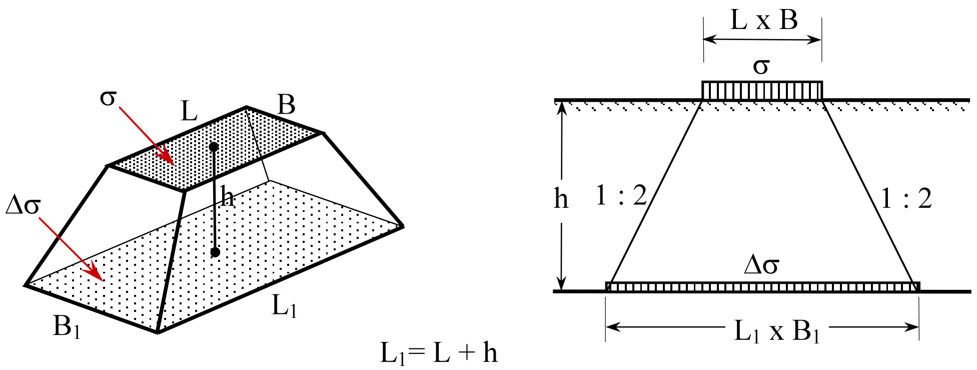


Fig 7.7 Use of Newmark chart

7.5 Approximate method

The approximate method gives average stress at the required depth due to load uniformly distributed over an area. The method assumes that the load is distributed in the soil at surfaces sloping 1 horizontal: 2 vertical, Fig. 7.8.



$$L_1 = L + h$$
$$B_1 = B + h$$

$$\boxed{\Delta\sigma = \sigma \cdot \frac{L.B}{L_1.\,B_1}}$$

Fig 7.8 Approximate method for stress distribution in soil

7.6 Contact stress

Contact stress is defined as the vertical stress acting at the surface of contact between the base of a foundation and the underlying soil mass. To simplify design, the computation of the bending moments, etc.. in the footing are commonly based on the assumption that the contact stress is uniform. The actual contact stress, however, depends upon the rigidity of the footing and the elastic properties of the subgrade (soil).

If the foundation is flexible, the distribution of the contact stress is uniform irrespective of the type of the subgrade or underlying soil material. If the footing is perfectly rigid, the contact stress distribution depends on the type of subgrade. Fig. 7.9 shows the stress distribution under flexible and rigid foundations resting over real elastic material, such as saturated clay, cohesionless sand and soil having intermediate characteristics. In case of clay, the theoretical value at the center is q/2, and infinite at the edges. However, due to local yielding the infinite value is reduced to a definite value. In the case of sand, no resistance to deformation is offered at the edges making the contact stress zero. With the increase of foundation width, the distribution tends to be uniform.

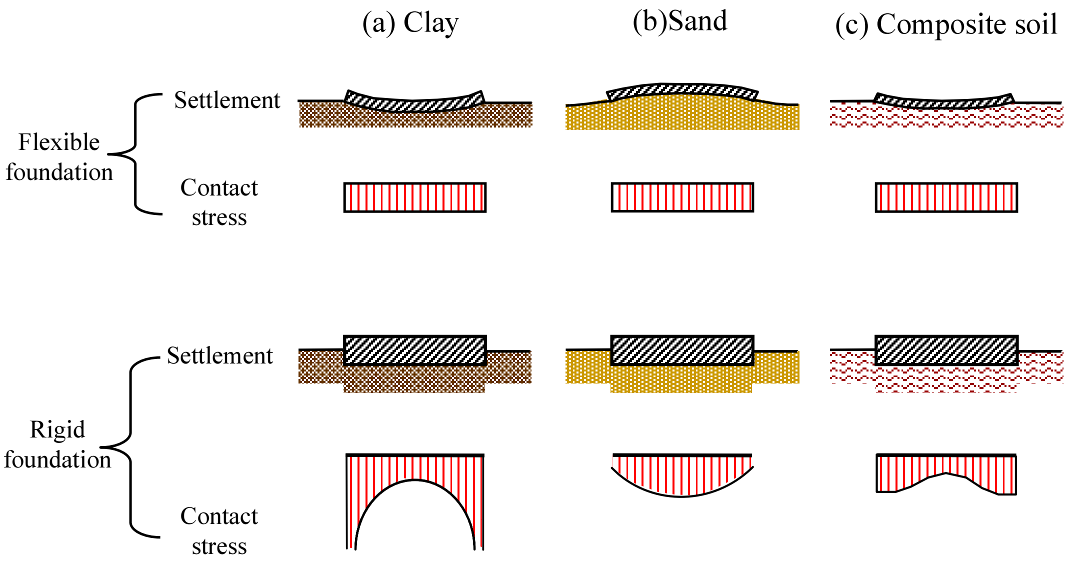


Fig. 7.9 Contact stress distribution

7.7 Examples

(1) A concentrated load of 10 t acts on ground surface. Use Boussinesq's equation to draw the isobars (lines of equal stress) corresponding to stresses 8 and 3 t /m^2.

Solution:

$$\sigma_z = \frac{Q}{z^2} \cdot N_B \qquad \text{(see Fig. 7.2 for } N_B \text{ values)}$$

$$\text{At } r = 0 \qquad N_B = \frac{3}{2\pi} = 0.477$$

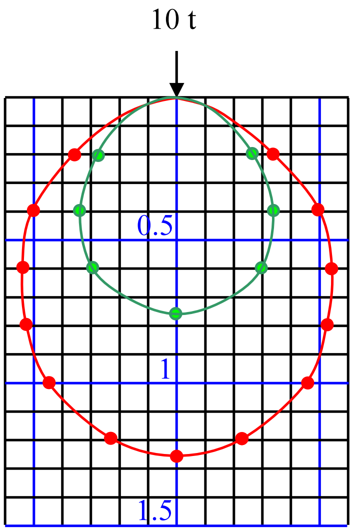
$$\text{For } \sigma_z = 8 \text{ t/m}^2 :$$

$$8 = \frac{10}{z^2} \times 0.477 \quad \therefore z = 0.77 \text{ m} \quad \& \;\; N_B = 0.8\, z^2$$

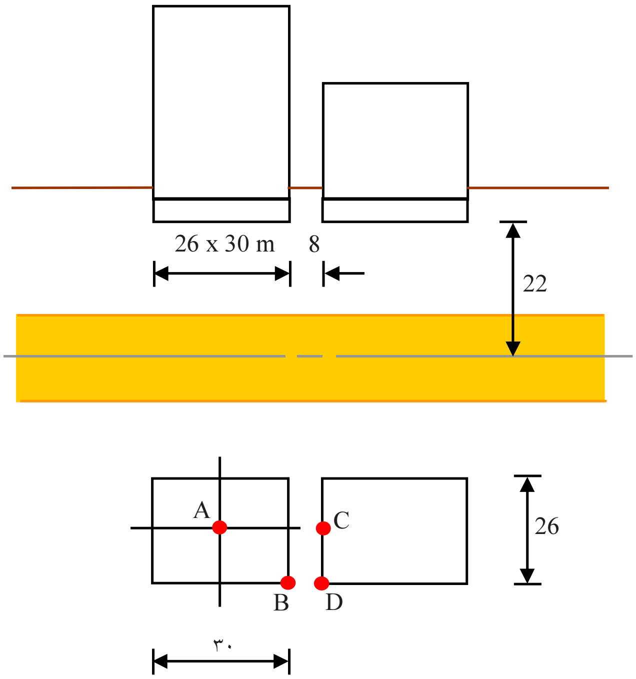
$$\& \;\; \text{For } \sigma_z = 8 \text{ t/m}^2 :$$

$$3 = \frac{10}{z^2} \times 0.477 \quad \therefore z = 1.26 \text{ m} \quad \& \;\; N_B = 0.3\, z^2$$

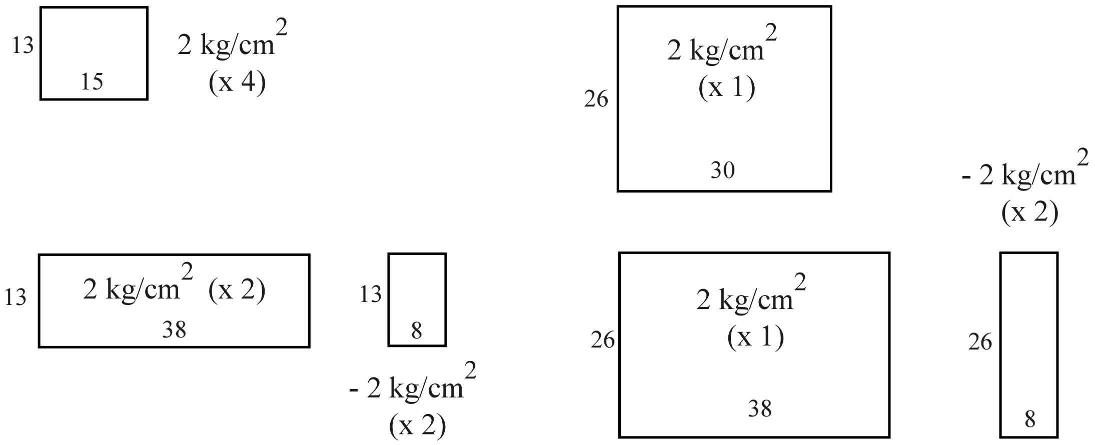
z m	$\sigma_z = 8$ t/m^2			$\sigma_z = 3$ t/m^2		
	NB	r / z	r	NB	r / z	r
0.2	0.032	0.716	0.279	0.011	0.545	0.366
0.4	0.128	1.201	0.333	0.048	0.814	0.49
0.6	0.288	2.112	0.284	0.108	1.109	0.54
0.8				0.192	1.508	0.53
1.0				0.299	2.212	0.451
1.2				0.431	4.948	0.242



(2) The figure shows a new building which exerts a stress of 2 kg/cm^2 at foundation level. Calculate the increase of stress due to the new construction at the centre line of the shown clay layer under points A, B, C and D. Assume that the soil is a semi-infinite homogenous isotropic elastic media.

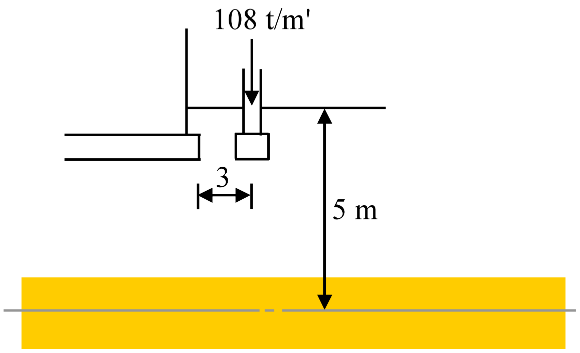


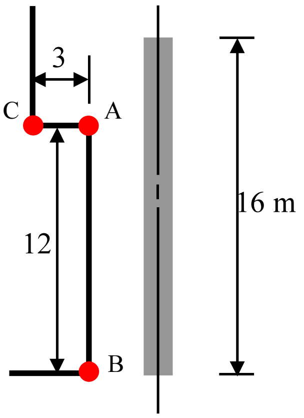
Solution:



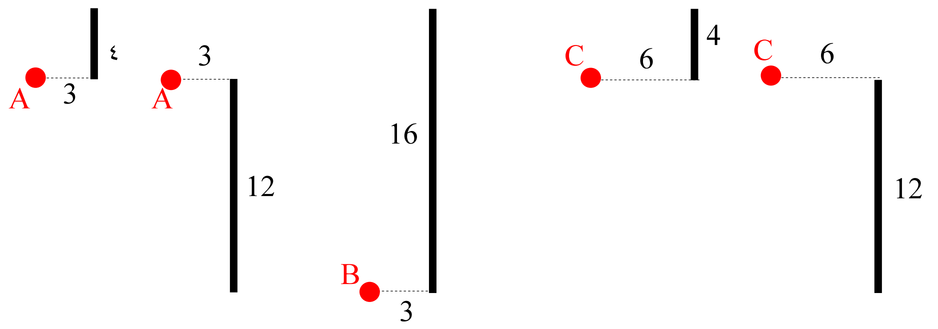
Point	q	a m	b m	m' a/b	n' z/b	N_r	$(\sigma_z)_i$	Num. repet.	$\Sigma(\sigma_z)_i$	σ_z kg/cm^2
A	2	15	13	1.15	1.69	0.114	0.228	4	0.912	0.912
B	2	30	26	1.15	0.85	0.193	0.387	1	0.387	0.387
C	2 -2	38 13	13 8	2.92 1.625	1.69 2.75	0.149 0.073	0.6 -0.294	2 2	0.6 -0.294	0.306
D	2 -2	38 26	26 8	1.46 3.25	0.85 2.75	0.208 0.09	0.417 -0.196	1 1	0.417 -0.196	0.221

(3) The figure shows an R.C. wall, 16 m long carries a load of 108 t/m'. The wall is built 3 m far from an existing building. Calculate the stress increase due to the wall at the center line of the shown clay layer below points A, B and C of the building. Neglect foundation depth effect. Assume that the soil is a perfect elastic media.



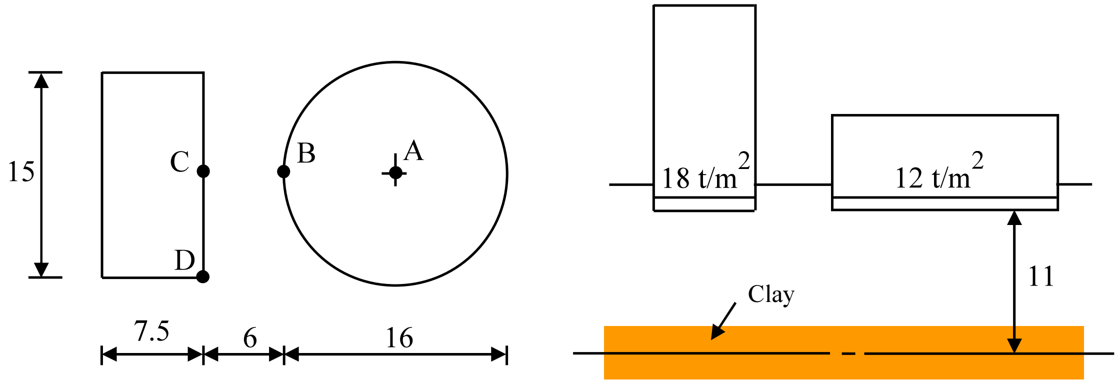


Solution:



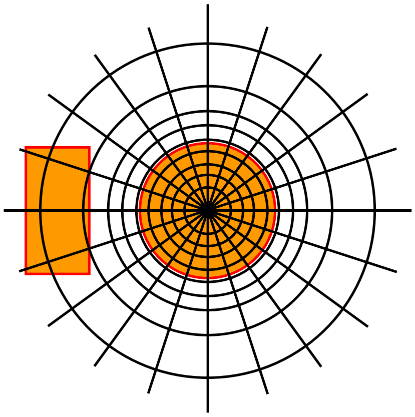
Point	q t/m'	x m	y m	m x/z	n y/z	N_L	$(\sigma_z)_i$ t/m'	Num. Repet.	$\Sigma(\sigma_z)_i$ total t/m^2	σ_z total t/m^2
A	108	3 3	4 12	0.6 0.6	0.8 2.4	0.125 0.172	2.7 3.71	1 1	2.7 3.71	6.41
B	108	3	16	0.6	3.2	0.178	3.85	1	3.85	3.85
C	108	6 6	4 12	1.2 1.2	0.8 2.4	0.033 0.048	0.71 1.04	1 1	0.71 1.04	1.75

(4) Two buildings are to be constructed as shown in figure. The stresses at foundation level are given. Find the increase of stress at the center line of the shown clay layer below points A, B, C and D due to the construction of both buildings. Assume that the foundations are flexible, and that the soil is a semi infinite elastic media.



Solution:

Considering the distance AB of the Newmark chart, the scale will be : 11 m = 1.5 cm. The plan is draw in this scale and the points A, B, C and D are placed at the center of the chart. Number of divisions N inside the circular area on which a stress of 12 t/m^2 acts is counted, and number of divisions N inside the rectangular area at which a stress of 18 t/m^2 acts is counted.

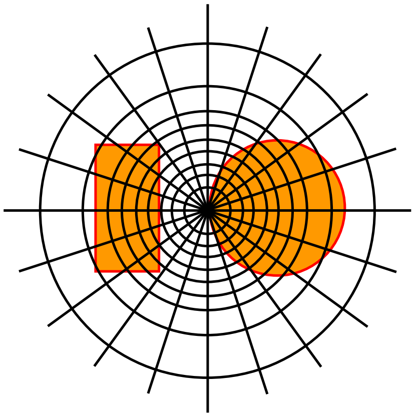


$N_{circle} = 90$

$N_{rectangle} = 4$

$\sigma_A = (90 \text{ x } 12 + 4 \text{ x } 18) \text{ x } 0.005$

$= 5.76 \ t/m^2$

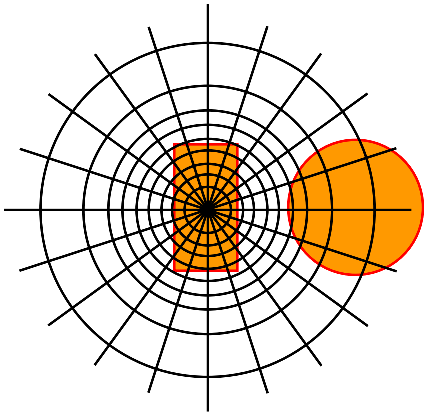


N_{circle} = ٥١

$N_{rectangle}$ = ١٩

σ_A = (٥١ x 12 + ١٩ x 18) x 0.005

= ٤.7٧ t/m^2

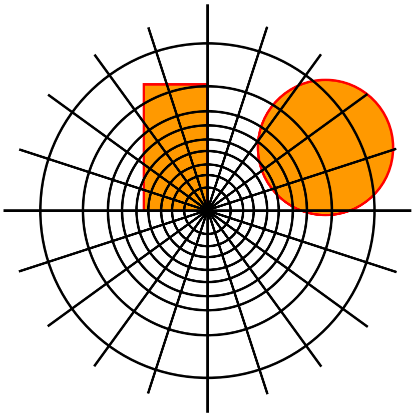


$N_{circle} = 19$

$N_{rectangle} = 47$

$\sigma_A = (19 \text{ x } 12 + 47 \text{ x } 18) \text{ x } 0.005$

$= 5.37 \ t/m^2$



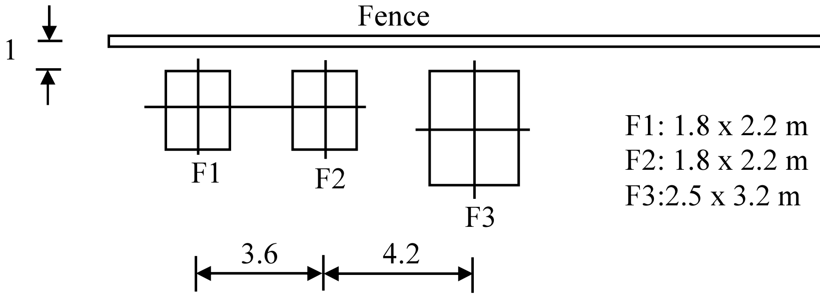
$N_{circle} = 13$

$N_{rectangle} = 30$

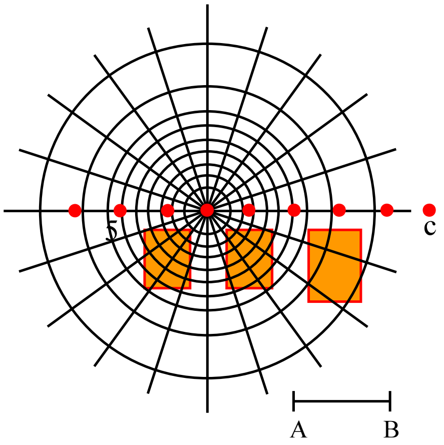
$\sigma_A = (13 \text{ x } 12 + 30 \text{ x } 18) \text{ x } 0.005$

$= 3.48 \ t/m^2$

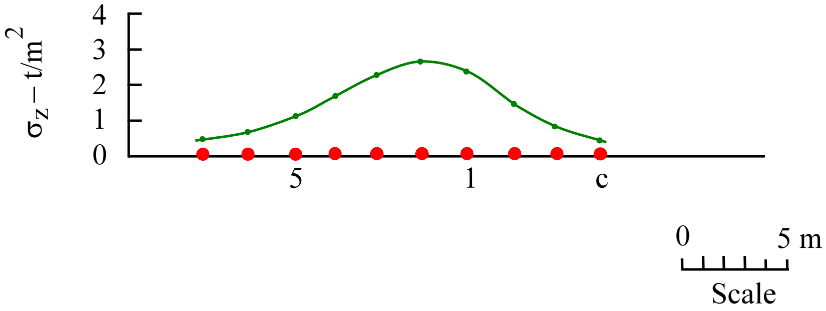
(5) A new building is constructed adjacent to a fence. Draw the stress distribution at 4 m depth below the fence due to stresses of 2.5 t/m^2 exerted by the shown three footings of the new building. Assume that the fence foundation and footings are at ground surface.



Solution:

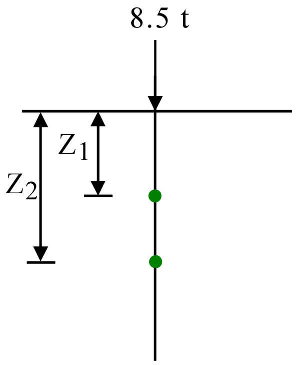


Using the shown Newmark chart, the scale will be 4 m = AB. The plan is drawn to this scale. Divide the fence into equal distances by points 1, 2, 3, etc. a, b, c, etc. Place each point at the center of the chart and count the number of divisions. Since the stress is equal on all loaded areas, find the total number of divisions N and use it values of stresses (σ_z) as ordinates. Join points as smooth curve giving the required distribution.

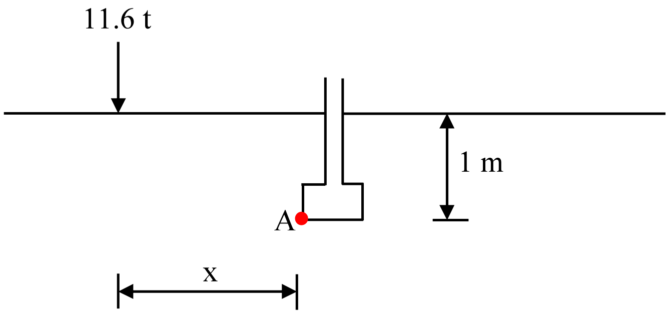


7.8 Problems

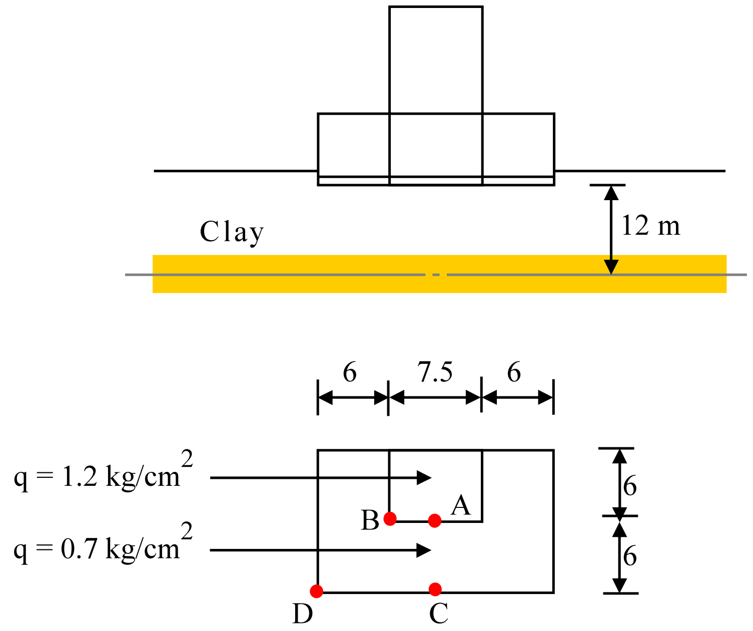
(1) A concentrated load of 8.5 t acts on ground surface. Use Boussinesq's equation to calculate the depths z_l and z_2 below the point of application of the load at which the values of stress increase due to the load are 1 and 0.5 t/m^2, respectively. Draw the isobars for these stresses.



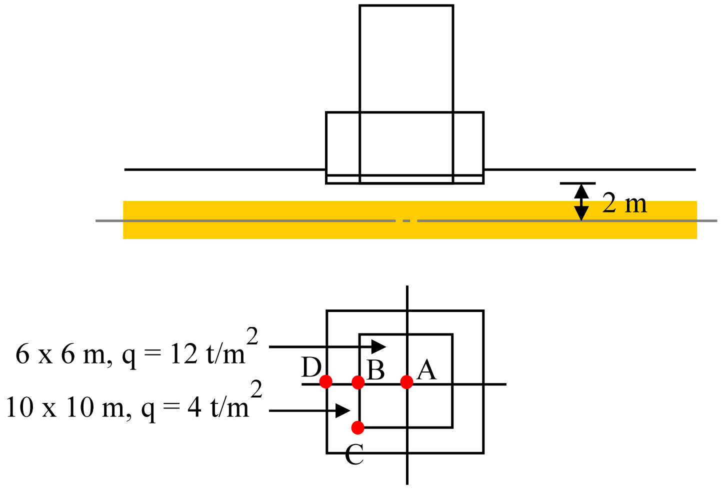
(2) The figure shows a concentrated load of 11.6 t acting on ground surface near a foundation. Find the shortest distance x such that the stress increase at paint A due to the concentrated load does not exceed 2 t/m2.



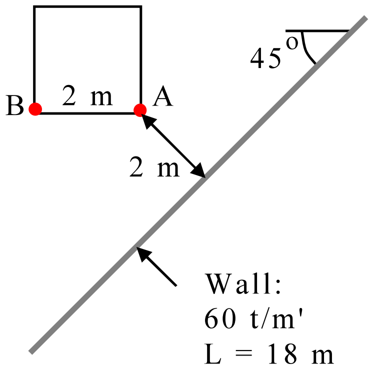
(3) The figure shows a building exerting a stress of 1.2 kg/cm^2 at the foundation of its higher part, and a stress of 0.7 kg/cm^2 at the foundation of its lower part. Find the stress increase at the centre line of the shown clay layer below points A, B, C and D, due to the building loads.



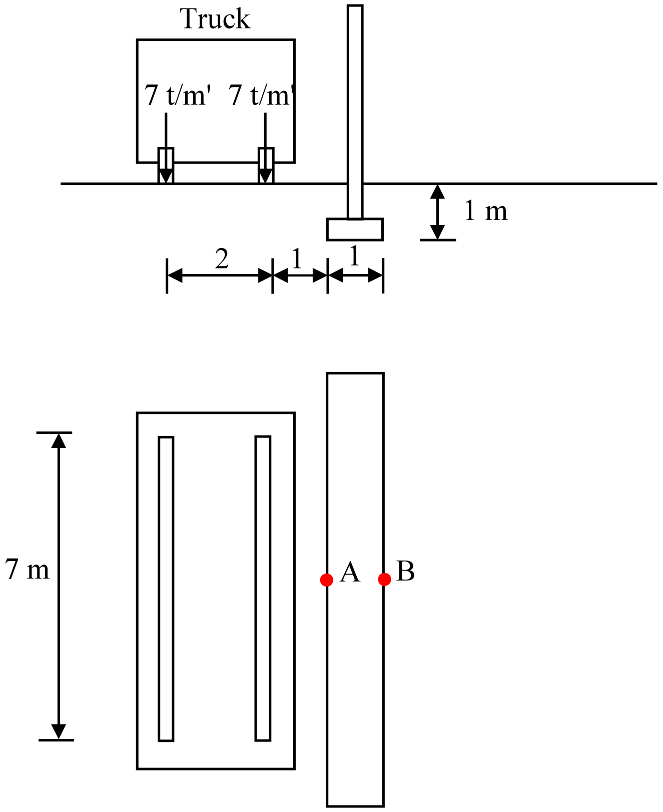
(4) Calculate the stresses below points A, B, C, D and E at the centre line of the clay layer due the shown construction.



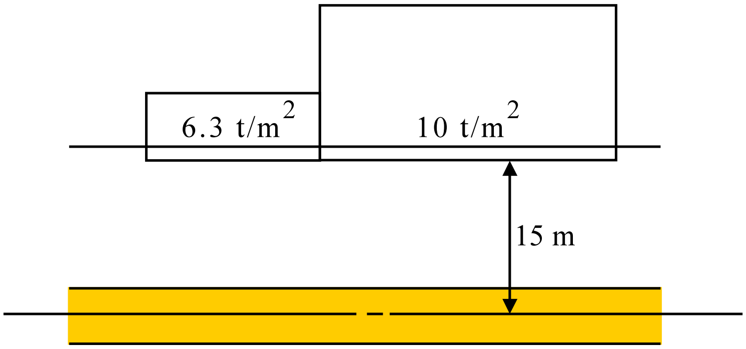
(5) A line load of 6o t/m' acts on the shown wall. The soil strata includes a clay layer with its centre line 2 m below the foundation of the line load. Find the stress increase below points A and B at a nearby building due to the line load.

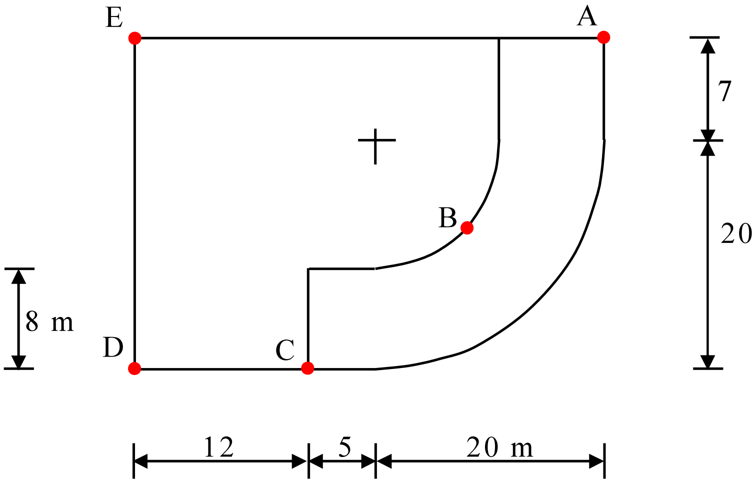


(6) Calculate the vertical stresses at points A and B of the shown foundation due to the shown truck loads.

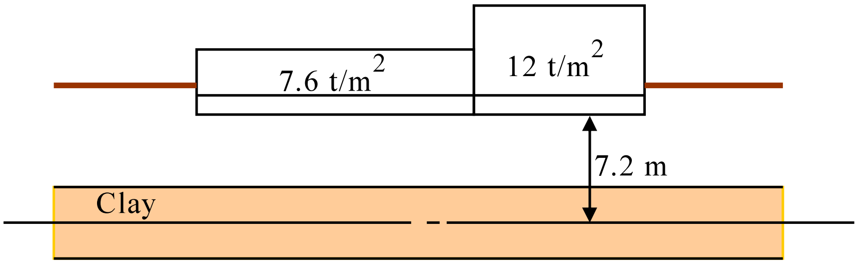


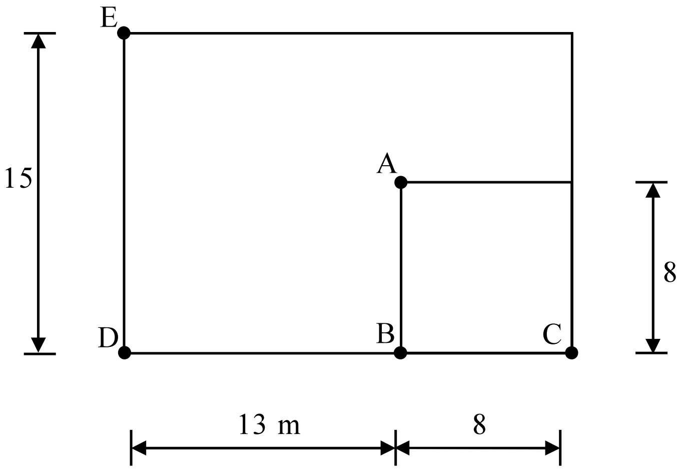
(7) The figure shows a building exerting a stress of 10 t/m^2 at the foundation of its higher part, and a stress of 6.3 t/m^2 at the foundation of its lower part. Find the stress increase at the centre line of the shown clay layer below points A, B, C, D and E, due to building loads.





(8) The figure shows a building exerting a stress of 12 t/m^2 the foundation of its higher part, and a stress of 7.6 12 t/m^2 at the foundation of its tower part. Find the stress increase at the centre line of the shown clay layer below points A, 8, C, D and E, due to building loads.


7.6 t/m2
12 t/m2
7.2 m
Clay


E
A
15
8
D
B
C
13 m
8

* * *